2. GRAPHING CALCULATORS AND THEIR POTENTIAL FOR TEACHING AND LEARNING STATISTICS

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GRAPHING CALCULATORS: AN OVERVIEW

The world today is described as a world based on information (National Council of Teachers of Mathematics, 1989), and reports on the rapid increase of information use figures such as “doubling every four years “or “increasing exponentially. Technology is not only responsible for producing much of this information, it is a critical tool in the way information is analyzed. Processing information often falls into the domain of statistics, and, although statistics has recently become a part of the mainstream curriculum in the United States, lessons are often focused on simple plots and finding standard measures of center, not on the task of processing information into useful and meaningful statements that can aid in understanding situations and making decisions. Recent developments in technology, including graphing calculators and statistics software packages with simulation capability, have the potential to transform the statistical content in the curriculum and how this content is taught.

In general, the potential for graphing calculators to radically change the teaching of mathematics is enormous. On a voluntary basis, secondary teachers in the United States have embraced them as an exciting and useful tool for the classroom. Hundreds of workshops are given each year, usually by teachers teaching other teachers, where participants learn to use the spreadsheet functions, graphing capabilities, and the programming logic of the calculators. The secondary mathematics curriculum has begun to reflect the changes made possible by the calculator; for example, students study functions in great detail, collect and analyze data from scientific experiments, and use programs to do complicated sorting and analyses. These changes also have an affect on the statistics curriculum.

Technology makes statistics and statistical reasoning accessible to all students. Students can analyze data numerically and graphically, compare expected results to observed results, create models to describe relationships, and generate simulations to understand probabilistic situations in ways that would not be possible without technology. Technology allows students to use real data in real situations. It also allows students to move easily between tabular representations, graphical representations, and symbolic representations of the data, and provides the opportunity to think about how each representation contributes to understanding the data. Students learn to recognize that considering either number summaries or graphical representations alone can be misleading.

The plots in Figure 1 were created from a dataset generated by John McKenzie from Babson College. Number summaries alone of these data are misleading; in each case, the mean is 50 and the standard deviation (SD) is 10. Graphs alone can also be misleading because of scale differences or modifications (de Lange, Wijers, Burrill, & Shafer, 1997). Technology makes it possible for students to encounter examples where graphs and numerical summaries or symbolic representations are used together to create a picture of the data.

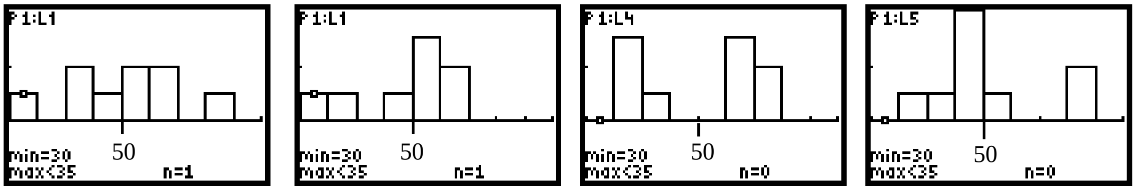


Figure 1: Four possible histograms where mean = 50 and SD = 10

In the past, students produced numerical summaries such as the mean, mode, or range. Descriptions consisted of statements similar to: "The mean wage was $300 per week. Most people earned $250." Making plots was a tedious task, and calculating the SD seems to have been considered so complicated that it was not taught in the United States until students were nearly finished with their formal schooling, if ever. Consequently, students had little experience with variability and understanding its importance, with inspecting distributions and understanding how they can have the same characteristics yet be very different, with looking at alternative displays of the same data and understanding how they each reveal something new about the data, or learning enough about the sampling process to trust conclusions based on random samples. Technology makes these ideas accessible.

Technology not only helps students analyze data, it also enables students to grapple with and develop their own understanding of important ideas. Students can estimate the ages of famous people and create a scatterplot of estimated age and actual age. To answer the question, "Who is the best estimator?" students must quantify their thinking (e.g., by counting the number correct, minimizing the total difference between the estimate and the actual value, or overestimating and underestimating in equal measures). Students confront the reality of outliers and discuss the appropriate choice for a summary number. They come to understand the impact of their choices on their final decision--much the way statistics is used in making other, and more important, decisions.

Technology allows students to do old things in new ways that enhance concept development and student understanding. Technology also allows students to do things that were not possible before. The focus of this paper is on five areas in statistics at the secondary level that have been affected by technology in either of these ways (i.e., doing old things new ways or doing new things): introductory data analysis, linear equations, least squares linear regression, sampling distributions, and multivariate regression. The discussion assumes that students have had some hands-on experience tossing coins or dice or sketching plots to help them understand the process before they use the technology. The problems were developed using a graphing calculator, but computer software packages can do the same analysis. The advantage of the calculator is that every student can have one for use at any time and at any place at a much lower cost.

A classroom experiment

The following lesson can be done in a class where every student or pair of students has a graphing calculator. It exemplifies the way using a graphing calculator can enhance student understanding and concept development. The lesson is a standard lesson that introduces single variable techniques through a class data collection activity focused around the question: How much change (pocket money) do people in class carry with them? As students report the amount of change they have in their pocket or purse, they enter the data for males and females into separate calculator lists that behave like a spreadsheet. By merging files, students can produce a histogram (see Figure 2) of the data and think about the representation before they begin to calculate numerical summaries.

In this example, students observe that the data are skewed, with most people having less than $.50 in change with them. There appears to be at least one extreme--someone who had between $7.00 and $7.50. By using the cursor as a balance point (see Figure 3), students can estimate the mean amount of change. They can also estimate the percent of people who have less than $1.00 (approximately 25%) or more than $5.00 (approximately 10%). By thinking this way, students are developing a foundation for thinking about area as a measure of probability.

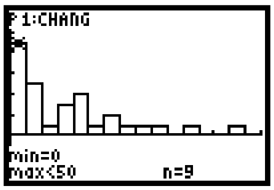


Figure 2: Class change data

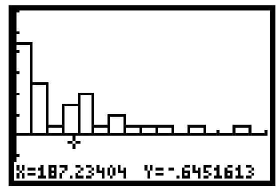


Figure 3: Cursor as balance point

The mean (181.63) and median (136) are quite far apart, which should cause students to consider how this was represented in the distribution and about the variability in the data that this difference represents. Students might be asked to experiment with data and plots using their calculators to produce a distribution where the mean and median are the same.

Before spreadsheets, students pushed a button that produced the SD, a number not well understood and so tedious to find by hand that the meaning got lost in the calculations. Students can now use list functions to find the “typical” difference in the amount of change from the mean of $1.81. Figure 4 shows that each individual difference is calculated in L2. Note that although the calculator does the work, the students see the results.

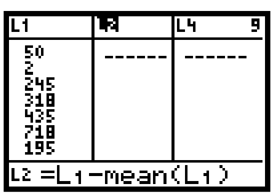


Figure 4: Screen display

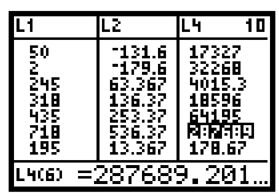


Figure 5: Screen display

The list displays negative differences (see L2 in Figure 5); when students find these differences by hand, they tend to ignore order and use the positive difference. When the list is squared to eliminate the negative (L4), an inspection of the values reveals the largest squared difference of 287,689, which was produced by the outlier of $7.18. (It is important to have each person identify their own “squared difference” to make this point very meaningful.) The contribution of this large value to the typical difference from the mean is apparent. Without actually being told, students are thinking about variability and SD. Once students have had some informal experience with how the SD is calculated and what it represents, they can be introduced to the symbol for standard deviation (σ) and value calculated in the stat calc menu (see Figure 6). They can also explore a formal definition of the term “outlier” to determine whether $7.18 is actually an outlier. A boxplot of the data (Figure 7) shows a five-point summary of the amount of change carried by the class.

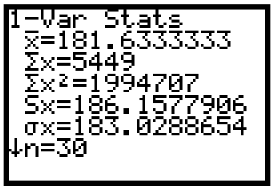


Figure 6: Stat Calc menu

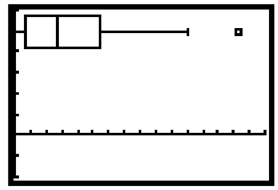


Figure 7: Boxplot to demonstrate an outlier

Technology allows students to experiment with data. They can estimate the impact on the average squared difference (variance) if the outlier is removed, then check the actual result by altering the lists and recalculating. (A replay key on the calculator makes this very simple to do.) Students can investigate relationships between different categories within the data. To investigate whether there is any difference in the amount of change carried by males and females, the students can create parallel boxplots (see Figures 8 and 9). Technology allows students to move from constructing plots to thinking about the information that the plots convey. Students can consider questions, such as (1) Describe the difference between the amount of change carried by males and by females; (2) What do you think a histogram of the amount of change for females would look like? (3) Why is the $7.18 not an outlier in the box plot for the change carried by females?

Students can transform the data (e.g., by giving $.50 to each person or by assessing a 10% tax) and investigate the change in the statistical summaries and in the distributions. They can adjust the width of the intervals in the histogram to see how this changes the distribution and its interpretation. Students can investigate the connection between numerical summaries and graphical representations (using the cursor to estimate the mean), which can also be done without a graphing calculator. However, students then need access to a computer, otherwise the computations and plots must be done by hand. The sheer amount of time this involves precludes any investigation and often interferes with understanding. Technology provides students with new ways to think about data and to investigate options in describing and graphing data. It enables students to build a statistical “number sense.”

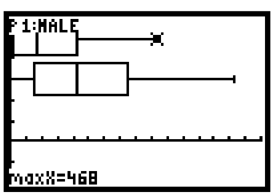


Figure 8: Male and female data

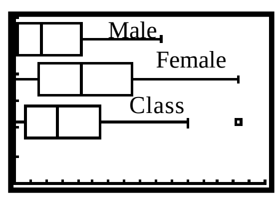


Figure 9: Adding class data

Lines and algebra

Graphing calculators also enable students to explore linearity in a different way. In most traditional algebra work, students explore lines that are determined: "Given two points, write the equation of the line" or "Find the equation of the line graphed in the plot." Because students can easily plot actual data when they are using a graphing calculator, the introductory work with the equations of lines can be with real data and lines that are not predetermined (Burrill & Hopfensperger, 1997). The plots illustrate the difference between a deterministic equation, given by $y = 3x + 8$ (Figure 10), and a data-driven equation that could take on several equally legitimate forms that each describe the relationship between the calories in certain fast foods and the amount of calories from fat in those foods.

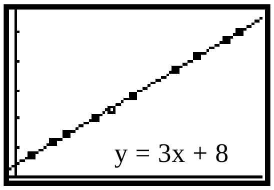


Figure 10: $y = 3x + 8$

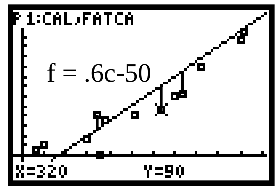


Figure 11: $f = .6c - 50$

Students can fit different lines and think carefully about the criteria for determining a "good fit." A toothpick can be placed on the points depicted on the screen of a calculator to find a variety of linear equations (see Figure 11). The slope of an equation has meaning within the context; for example, the slope of .6 would represent an increase of 6 calories from fat for every increase of 10 calories. To check their line, students can find the difference between the actual number of calories and the number of calories predicted by the line for individual fast food items. Essentially, they are finding residuals to measure the "goodness of fit" of each line. Building on the work done with the SD, students can find the squared differences between observed values and predicted values (Figure 12). The list function allows students to quickly produce these calculations and find the sum of the squared residuals (Figure 13).

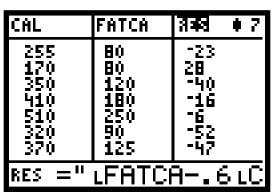


Figure 12: Squared differences

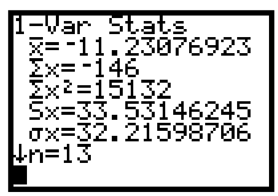


Figure 13: Sum of squared residuals

(Some students may choose to use the sum of the absolute residuals, which is appropriate at early levels; the difference between squared and absolute residuals can be explored as students' mathematical background is developed.) By experimenting with the slope and intercept, students can find equations that will produce smaller and smaller sums of squared residuals. The use of graphing calculators makes it possible to introduce, at least informally, statistical concepts such as the residual and the sum of squared residuals into standard mathematics content early in secondary school. Students can study lines and linearity in new ways, relate the slope to meaningful contexts, and begin to internalize the fact that the vertical distance between a point and a line describes a difference in the *y*-values.

Sampling distributions

In addition to exploring data in new ways, technology can affect how students come to understand the process of sampling. Random number generators are valuable tools in allowing students to explore probability ideas through simulation (Scheaffer, Swift, & Gnanadesikan, 1988). Coupled with list and sequence features, random number generators allow students to easily produce and explore sampling distributions. Students now have the opportunity to create sampling distributions for a statistic from a given population, which will help them understand that the behavior of a statistic in repeated samplings is regular and predictable. It is often difficult for students to understand that something that is random can, indeed, have regularity.

A typical class problem might be the following: Suppose that the percent of women in the workforce is 40%. About how many women would you expect to see in a sample of 30 randomly selected workers? Using the random binomial generator, students can quickly compile a sampling distribution of 50 samples of size 30; this can be done repeatedly (Figure 14).

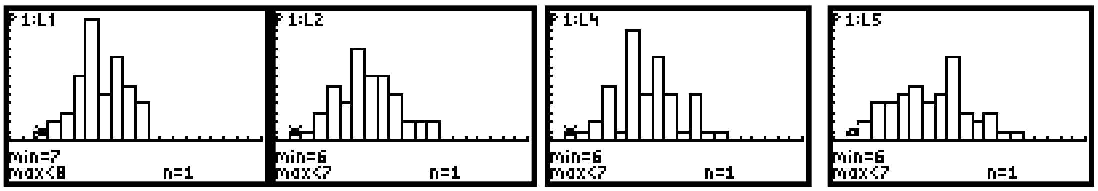


Figure 14: Sampling distributions

Initially, the results of the repeated sampling do not look similar, but some general observations can be made. For example, to find less than 6 or more than 19 women workers in a sample of 30 seems highly unlikely, and the mean or balance point is approximately 11 or 12 in each distribution. Students can construct 90% boxplots, which are boxplots that contain at least 90% of the outcomes. These can then be compared for the different sets of samples (see Figure 15).

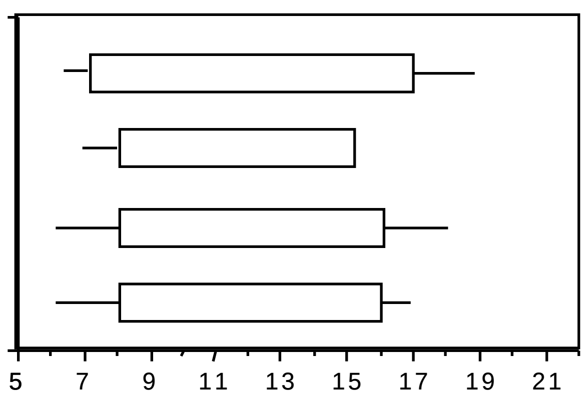


Figure 15: 90% boxplots for samples of size 30, p = .4

The transition from a specific context (women workers) to a general pattern is not obvious to students. Initially, many students treat each situation as a separate problem and carefully reproduce the simulation, but as they continue to investigate comparable situations, they begin to make some generalizations. They gradually recognize that the following two problems are essentially the same as the women in the workforce problem:

40% of the people in the community subscribe to the daily newspaper. In a sample of 30 people, how many newspaper readers would you be likely to find?

40% of those who marry before the age of 25 are divorced by age 40. In a sample of size 30 from this population, how many divorced people would you be likely to see?

As the contexts change, students can see that the outcomes depend only on the population proportion and the sample size (Hirsch et al., in press). They learn that changing the population proportion yields a very different distribution (Figure 16) and recognize the effects on the distribution of increasing the sample size (Figure 17). (For samples of size 30, the likely results span approximately 11 out of 31 possible outcomes; for samples of size 50, the likely results span 14 out of 51 outcomes; the proportion of the total range decreases as the sample size increases.) Students can also generate the distributions for an increasing number of samples and see how the results converge to an expected shape for the distribution and to an expected mean (Figure 18).

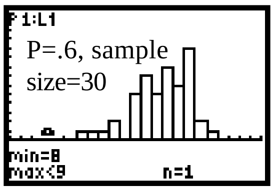


Figure 16: Sampling distribution for p = .6, sample size = 30

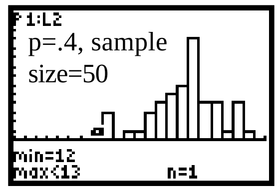
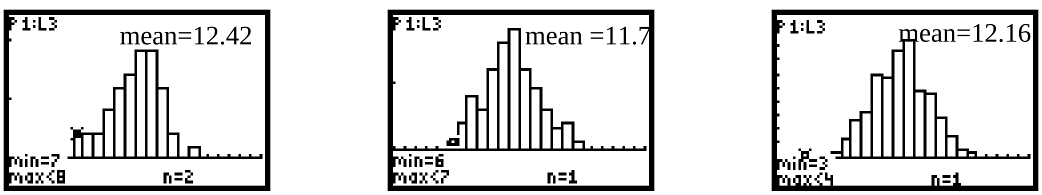


Figure 17: Sampling distribution for p=.4, sample size = 50



50 trials 100 trials 500 trials

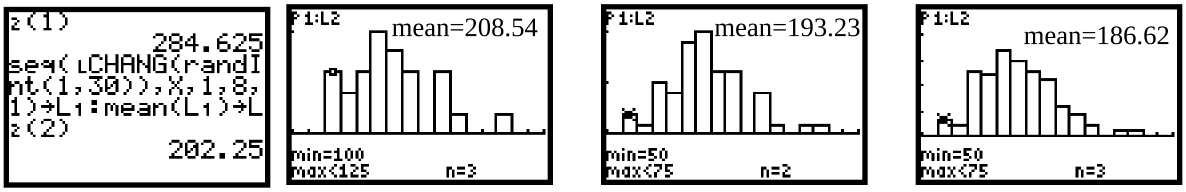
Figure 18: Sampling distributions for p = .4, sample size = 30

As background for the central limit theorem, students can continue to investigate the sampling process by drawing samples from a given population and studying the characteristics of the resulting distributions. The calculator routine involves the sequence command as well as the random number generator. Consider the pocket change example presented above. The calculator will randomly select 8 people from the list, calculate the mean amount of change for those 8, and store the mean in a new list (L2). Figure 19 shows this process and the distribution as the number of samples (of size 8) increases from 25 to 50 to 100. The mean of the first sample is stored in L2(1), the second in L2(2), and so on. The first command is:

seq(⌊Chang(randInt (1,30)), x, 1, 8 , 1) -> L1: mean(L1) -> L2(1).

Using the replay key to change the storage position for the mean from the second sample yields:

seq(⌊Chang(randInt(1,30)),x,1,8,1) -> L1: mean(L1) -> L2(2), and so on.
(Note that the mean of the original data is 181.6.)



25 samples 50 samples 100 samples

Figure 19: Distributions for samples of size 8

A graphing calculator allows each student to observe the results of the random sampling process. They observe that patterns do exist and learn to trust that the process will, if done properly, always produce these patterns. Conclusions based on random sampling can be quantified with some degree of certainty and can be trusted to paint the general picture of a situation. Simulations and this kind of reasoning can also be used to develop student understanding of confidence intervals (Landwehr, Swift, & Watkins, 1987).

Least squares regression

Technology allows us to teach statistics at the secondary level that in the past was not part of the curriculum. Graphing calculators have provided a platform for working with paired data in many new ways. Calculators that have menus with regression models to describe relationships in paired data have changed not only the statistics taught to students in secondary schools but also the content of mathematics courses and texts. Almost every current text that deals with algebra has a section on curve fitting, unfortunately often done in inappropriate situations with little attention paid to underlying concepts. Students are blindly fitting models to data, excited about obtaining better fits as the degree of the polynomial increases, with no understanding that for eight data points, a seventh degree polynomial will fit exactly. The mathematical understanding necessary to understand the process is lagging behind the power of the technology .

Because they integrate tables, graphical representations, and number summaries, graphing calculators can be used effectively to help students understand what least squares linear regression is and how it behaves. Table 1 contains information on the top 10 films for the weekend of February 28 to March 1, 1992 from *Variety* (Burrill, Hopfensperger, & Landwehr, in press) The "box office revenue" column is the amount of money that the movie grossed in units of $10,000. Figure 20 shows a plot of the data..

Students explore finding an equation to model a linear relationship between the number of screens and box office receipts using the smallest sum of squared residuals as their criterion. They can fix the point (mean x, mean y) or (1418 screens, 375 $10,000 in receipts) and investigate the sum of squared residuals for slopes of different possible lines containing this point (Figure 21). This can be done easily by using the list labels and changing just the slope, and recording the corresponding sum of squared residuals. Note that a slope of .3 reduces the sum of squared residuals to 5,927,343 (Figure 22).

As a group, students can input the slopes and sums of squared residuals on their calculators and plot the data (Figure 23). The quadratic pattern is usually a surprise. Students use their knowledge of parabolas to find the minimum point and, thus, the slope that will have the least sum of squared residuals (Figure 24). According to this investigation, a likely candidate for the line that produces the least sum of squared residuals would have a slope of .33 and contain the point (1418, 375): B = 375 + .33(S-1418) where B is box office receipts in $10,000 and S is the number of screens.

Table 1: Movie income

Film	Number of Screens	Box Office Revenue (x $10,000)
Wayne's World	1878	964
Memoirs of an Invisible Man	1753	460
Stop or My Mom Will Shoot	1963	448
Fried Green Tomatoes	1329	436
Medicine Man	1363	353
The Hand That Rocks the Cradle	1679	352
Final Analysis	1383	230
Beauty and the Beast	1346	212
Mississippi Burning	325	150
The Prince of Tides	1163	146

Source: *Entertainment Data Inc.* and *Variety*, 1992

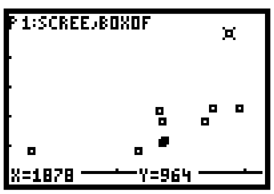


Figure 20: Scatterplot

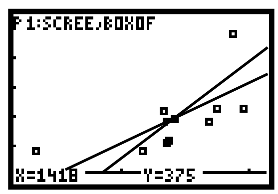


Figure 21: Possible fitted lines

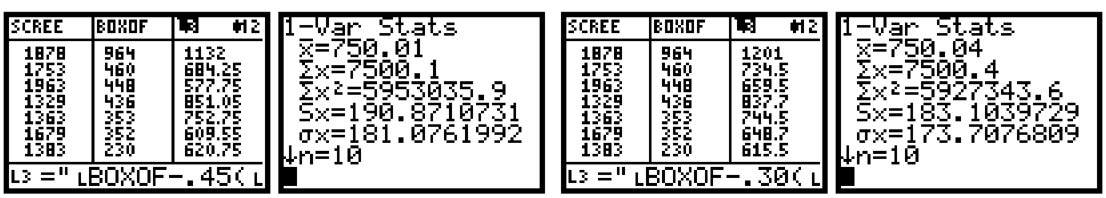


Figure 22: Screen displays for squared residuals

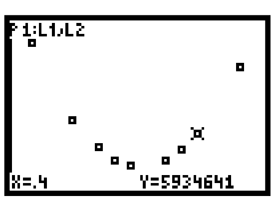


Figure 23: Data plot

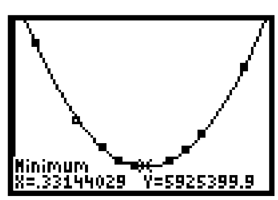


Figure 24: Minimum point

Students can continue the investigation by fixing the slope and varying the point. With appropriate graphing software, they can also vary both the slope and intercept to produce the least sum of squared residuals in a three-dimensional plot. A graphic representation of the least squares regression line can be found by using dynamic geometric software. As students change the slope or the *y* intercept, the squares change size, and the table values will produce the sum of squares for each new line (Figure 25). The least squares regression line is thus the line that produces the smallest sum of squares.

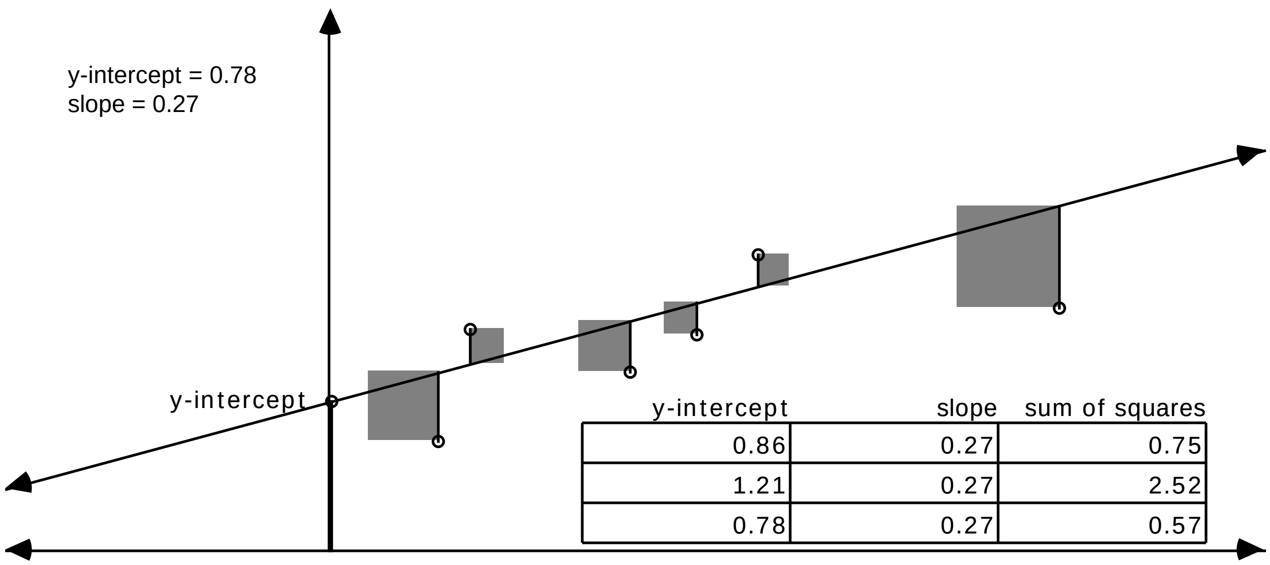


Figure 25: Least squares regression line

Correlation and graphs

A corresponding new topic in secondary statistics is the emphasis on correlation. The correlation coefficient is produced as a companion statistic to most of the regression models on the calculator. The integration of graphical representations and numerical calculations can provide students with real understanding of what the correlation coefficient indicates about a dataset--and what it does not. Correlation is a measure of the strength of the linear relationship between two variables. If knowing something about one variable helps you know something about the other variable, the correlation will be strong and close to either 1 or -1. There is a useful mathematical relationship relating the two variables that will help predict one from the other. If knowing something about one variable does not help explain the other, the correlation will be close to 0. In the long run, you will be better off using the average values instead of trying to use one to predict the other.

The critical factor in using correlation is to begin with the plot and decide first whether it is even reasonable to explore a linear relationship for the paired data. With calculators, it is easy to create the plot and look at the relationship before making any claims about the strength of r, the correlation coefficient. Table 2 provides the data used in Figure 26. This example is provided without a context, which strengthens the argument for being careful about correlation and helps students understand the power and the limitations of the correlation coefficient. The graphing calculator produces "r" or "r^2" automatically. Some thoughtful investigations are necessary to help students understand how to use r as an appropriate tool in the curve fitting process.

Table 2: Correlation data

I	II	III	IV	V	VI	IX	X
1.0	2.0	1.0	5.0	3	18	-4	16
1.5	2.3	2.0	9.0	5	15	-3	9
2.0	2.5	4.0	17.0	7	20	-2	4
1.4	0.3	6.0	25.0	8	24	-1	1
2.3	2.6	8.0	33.0	12	22	0	0
1.0	1.8	10.0	41.0	15	28	1	1
0.2	1.9	15.0	61.0	20	31	2	4
0.8	0.7	20.0	2.0	22	30	3	9
2.6	1.0	25.0	101.0	25	36	4	16
2.1	0.2						
30.0	45.0						

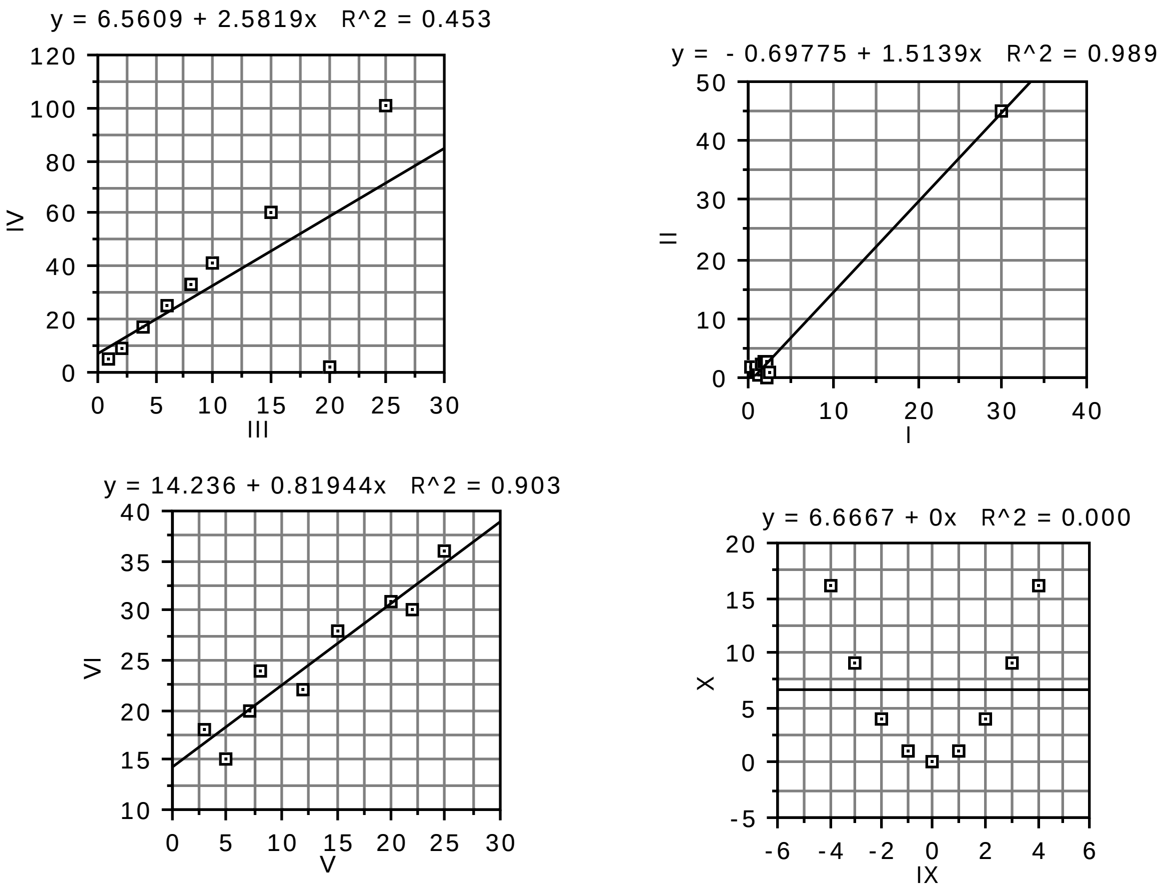


Figure 26: Correlation analysis

Multivariate regression analysis

Another example of an area of statistics that is now possible for students to learn because of access to graphing calculators is multivariate regression analysis. This topic has rarely been considered appropriate for secondary students, but is indeed manageable, and can be understood if the computations are carried out by a graphing calculator, particularly a calculator that converts list data into matrices. The Scholastic Achievement Test (SAT) is commonly administered in the United States to students entering the university from secondary school. The results are reported in two categories--mathematics and verbal--which can be used as possible indicators of student success at the university level (Witmer, Burrill, Burrill, & Landwehr, in press). The task is to use these two variables, SATM (the mathematics score) and SATV (the verbal score), to predict university grade point average (GPA) (see Table 3).

Table 3: College grade point average and SAT scores

Student number	GPA	SATV	SATM
1	3.58	670	710
2	3.17	630	610
3	2.31	490	510
4	3.16	760	580
5	3.39	450	510
6	3.85	600	720
7	2.55	490	560
8	2.69	570	620
9	3.19	620	640
10	3.50	640	660
11	2.92	730	780
12	3.85	800	630
13	3.11	640	730
14	2.99	680	630
15	3.08	510	610

Source: Oberlin College, 1993

Building on their knowledge of least squares regression, students can find the least squares model for (SATV, GPA) and find the sum of squared residuals for that model. The result is the predicted GPA in terms of the verbal score $\widehat{GPA}_V = 1.97658 + .001906V$ or in terms of the actual GPAs, the estimate plus the "error," $GPA = 1.97658 + .001906V + \mathbf{r_V} = \widehat{GPA}_V + \mathbf{r_V}$. The "error" or residual term, $\mathbf{r_V}$, represents the part of GPA that is not explained by V (the SAT verbal score) in the regression model.

It seems reasonable that GPA would depend on both SATV and SATM, probably with some additional error; that is, GPA = f(V) + h(M) + e for some functions f and h. Thus, f(V) = 1.97658 + .00l906V predicts GPA with error "$\mathbf{r_V}$." Think of h(M) + e as $\mathbf{r_V}$. This indicates that the residuals from using the verbal score to predict college GPA can be explained by M, the math score, or $\mathbf{r_V} = h(M) + e$. Thus, to find $\mathbf{r_V}$, fit a regression line to (SAT_M, $\mathbf{r_V}$), and you will get an equation predicting $\mathbf{r_V}$, $\hat{r}_V = -.52546 + .00083\,M$. Thus, combining both math and verbal scores to predict GPA:

$$
\begin{aligned}
\widehat{GPA}_{V\,M} &= 1.97658 + 0.001906V + \mathbf{r_V} \\
&= 1.97658 + 0.001906V + -.52546 + 0.00083\,M \\
&= 1.45112 + 0.001906V + 0.00083\,M
\end{aligned}
$$

However, the equation to predict GPA, $\widehat{GPA}_{V\,M}$, still has error: GPA = 1.45112 + .001906V + .00083 M + $\mathbf{r_{vm}}$. To explain $\mathbf{r_{vm}}$, you have only V and M as available variables. You can write $\mathbf{r_{vm}}$ as a function of V and find the regression for (SATV, $\mathbf{r_{vm}}$) to see whether this will improve your model by checking the sum of squared residuals. The iteration continues, with the sum of squared residuals decreasing until it eventually converges to the smallest sum of squared residuals possible.

To demonstrate the validity and power of the process, students can also begin with (SATM, GPA) and reach the same conclusions. This method is based on least squares linear regression. Another method involves matrices: The problem can be stated as $b_0 + b_1M + b_2V = GPA$ for coefficients b_0, b_1, b_2. Written as a matrix system this would be :

$$
[\,1\ \ V\ \ M\,] \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = [\,\widehat{GPA}\,] \quad \text{or } Xb = Y
$$

"Solving" the system using some knowledge of matrix procedures, particularly of the transpose, yields $\mathbf{b} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$. The SAT data can be moved from the list menu to matrices, and the formula for **b** yields:

$\widehat{GPA} = 1.528859829 + .00140995371M + .0011918744\,V$ (see Figure 27).

The matrix method is independent of the number of independent variables involved in making the prediction and gives students a procedure that can be used to find a regression model for any situation, as long as there is reason to suspect some correlation. Without the aid of the calculator, this would indeed be beyond the scope of students at the secondary level.

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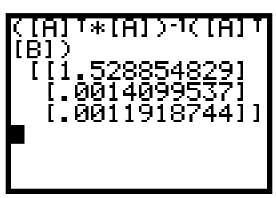

Figure 27: GPA equation

What can be eliminated

Some processes and procedures are no longer necessary because technology has made them obsolete. Alternate formulas created to help in the computation of certain statistics such as the SD or correlation are no longer critical. Certain computations such as Σ*xy* as tools for calculation are no longer of major importance. These formulas are useful for thinking about the mathematics underlying the relationships, but are no longer needed as part of the mainstream content. Technology that can sort and order allows median-based measures to assume roles in analysis as opposed to the past concentration on mean-based techniques, which were primarily used because of the relative ease of calculation. Previously, counting and sorting had to be done by hand and were almost impossible to do with large datasets. Because technology allows investigations of a variety of distributions, many of them discrete, it is no longer necessary to place as much reliance on the normal distribution and the many assumptions that must be made in order to use it correctly.

According to Rossman (1996), there are three basic uses for technology. First, technology can be used to perform calculations and present the graphical displays necessary to analyze real datasets, which are often large and use “messy” numbers. Second, technology can be used to allow students to conduct simulations, which let them experience the long term behavior of sample statistics under repeated random sampling. Third, technology enables students to explore statistical phenomena. Students can make predictions about a particular statistical property and then use a calculator to investigate the predictions, and then revise the predictions and iterate the process as necessary. Students can use calculators to investigate the best fitting lines, the effect of outliers, and the effect of sample sizes on confidence intervals. The goal of these uses, however, is to use statistics to turn information into knowledge. Statistics, enhanced by technology, can make the difference. In the words of T.S. Eliott (1971), "Where is the knowledge that is lost in information?" (p. 96).

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