12. WHAT DO STUDENTS GAIN FROM SIMULATION EXERCISES? AN EVALUATION OF ACTIVITIES DESIGNED TO DEVELOP AN UNDERSTANDING OF THE SAMPLING DISTRIBUTION OF A PROPORTION

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INTRODUCTION

The trend in statistics education over the past few years has been to replace extensive studies in probability, which lead to the theoretical development of the probability distribution, with computer-based simulation exercises that develop these ideas based on empirical results. This trend has been supported strongly by the statistics education community in general (e.g., Biehler, 1985; Gordon & Gordon, 1992; Hogg, 1992; Moore, 1992b) . In some cases, entire courses have been developed based on computer-based simulation (e.g., Martin, Roberts, & Pierce, (1994). Although this movement away from formal studies in probability could be well-founded, little formal research has been undertaken to determine the sort of understanding that develops as a result of such computer-based simulation exercises.

This study examined an area of statistics that has often been supported by computer simulation exercises--the development of the idea of a sampling distribution. This is a critical step in developing the theory of statistical inference; that is, the recognition that the estimates of a population parameter will vary and that this variation will conform to a predictable pattern. However, for all its importance, experience and research have shown that the idea is generally poorly understood (Moore, 1992a; Rubin, Bruce, & Tenney, 1990). One reason for this might be the way in which the idea has traditionally been introduced in statistics courses--by using a deductive approach based on probability theory (see e.g., Johnson & Bhattacharyya, 1987; Mendenhall, Wackerly, & Scheaffer, 1990) . Such explanations are usually expressed in a highly mathematical language that tends to make the argument inaccessible to all but the mathematically able, which comprises a very small minority of the students taking introductory courses in inferential statistics. But perhaps more importantly, it is a theoretical development that is difficult to relate to the physical process of drawing a sample from a population. Statistics educators have come to recognize that there are deficiencies with a purely theory-based explanation and often accompany or replace this with an empirical argument. This argument uses the relative frequency approach to probability, where the sampling distribution is viewed as the result of taking repeated samples of a fixed size from a population and then calculating the value of the sample statistic for each (Devore & Peck, 1986; Ott & Mendenhall, 1990) . The empirical approach has the advantages of being more readily related to the actual physical process of sampling and requires minimal use of formal mathematical language.

The computer has an obvious role in the empirical development of the idea of a sampling distribution. It is relatively easy to program a computer to draw repeated samples from a specified population and to then summarize the results. A number of instructional sequences have been developed built around these capabilities. Unfortunately, these approaches, although widely promoted and now commonplace activities in introductory statistics courses, may not have been as successful in developing in students the concepts of sampling distribution as statistics educators have hoped. For, as noted by Hawkins (1990).

> ICOTS 2 delegates were treated to "101 ways of prettying up the Central Limit Theorem on screen", but if the students are not helped to see the purpose of the CLT, and if the software does not take them beyond what is still, for them, an abstract representation, then the software fails. (p. 28)

What are the concepts we are hoping computer simulation exercises will clarify, and how does the nature of the computer strategy used facilitate the assimilation of these concepts? This paper examines some attempts at using computer-based strategies designed to introduce the idea of the sampling distribution empirically, and makes some preliminary evaluation of the understanding that may develop as a result of undertaking these experiences.

TWO COMPUTER-BASED STRATEGIES FOR INTRODUCING THE IDEA OF A SAMPLING DISTRIBUTION

In this evaluation, two computer-based instructional strategies that introduce the idea of a sampling distribution are considered. For convenience, we will restrict ourselves to the distribution of a sample proportion. The first strategy utilizes the general purpose computer package Minitab. The second strategy involves a computer package, Sampling Laboratory (Rubin, 1990), that explicitly makes use of the increased graphics potential of the new desktop computers.

Strategy 1 (Minitab)

In the early 1980's, the more innovative statistics educators began using the computer as part of their teaching sequence (e.g., Bloom, Comber, & Cross, 1986; Thomas, 1984). In the earliest attempts, complicated programming was required, but now commonly available statistical computer packages such as Minitab may be used to produce empirical sampling distributions. Students are given the appropriate computer code to generate random samples, calculate corresponding values of the sample proportion $\hat{p}$, and display the distribution graphically (generally in the form of a histogram). For example, students can instruct Minitab to draw 100 samples of size 25 from a population with proportion $p = .5$ and to construct a histogram of the 100 sample values of the sample proportion $\hat{p}$ (see Figure 1). Similarly, other histograms can be created by varying the population proportion p, the sample size n, or both.

Strategy 2 (Sampling Laboratory)

More recent computer applications in mathematics and statistics have tended to de-emphasize the use of the computer as a computational tool and to focus on our ability to use current technology to build a working model of the process under consideration and to display the results graphically (e.g., Shwartz & Yerushalmy, 1985). This approach has been followed in Sampling Laboratory (Rubin, 1990).

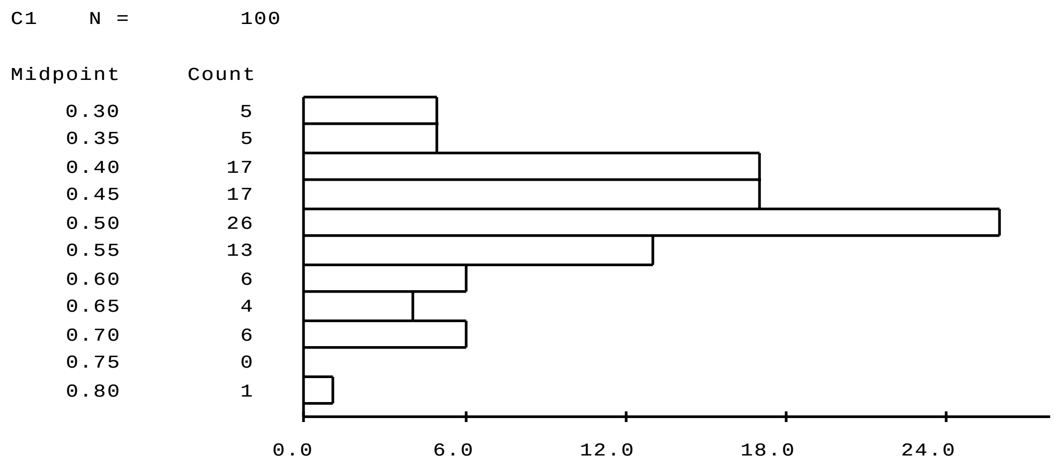

Figure 1: The histogram produced by Minitab

To use Sampling Laboratory, no programming is required by the user. The user simply makes the appropriate entries as requested. The required number of samples are drawn sequentially, and the screen shows simultaneously the following three windows:

1. A probability distribution/bar chart of the population proportion.
2. A bar chart that shows the number of outcomes in each category observed in the particular sample.
3. An empirical sampling distribution of the values of the sample proportion $\hat{p}$ that builds as the sampling proceeds, with the value of the sample proportion $\hat{p}$ from the last sample explicitly shaded. The overall sample proportion is also shown.

The screen after 30 samples have been drawn is shown in Figure 2. The same calculations that are performed in Minitab are also conducted here, but the emphasis is on the sampling process, and the calculations remain in the background. In Sampling Laboratory, the sampling process can be observed in real time and students see the sampling distribution form as more and more samples are drawn. The process may be stopped and restarted at any time, or may be conducted stepwise, one sample at a time.

THE STUDY

The objectives of the session were to introduce students to the following concepts:

- Samples will vary.
- As a consequence, the value of the sample statistic will vary from sample to sample.

- These sample statisics do not vary in a haphazard way but form a sampling distribution.
- This sampling distribution is roughly symmetric and bell shaped.
- The center of the sampling distribution is at the value of the population parameter.
- The spread of the sampling distribution is related to the sample size.

In order to evaluate the effect of using the computer simulation activities on students' understanding, an experiment was designed using students in an introductory statistics course at the university level in Melbourne, Australia. Students are graduates from a variety of courses, and some had taken courses in statistics previously, but for many this was their first experience studying any quantitative discipline.

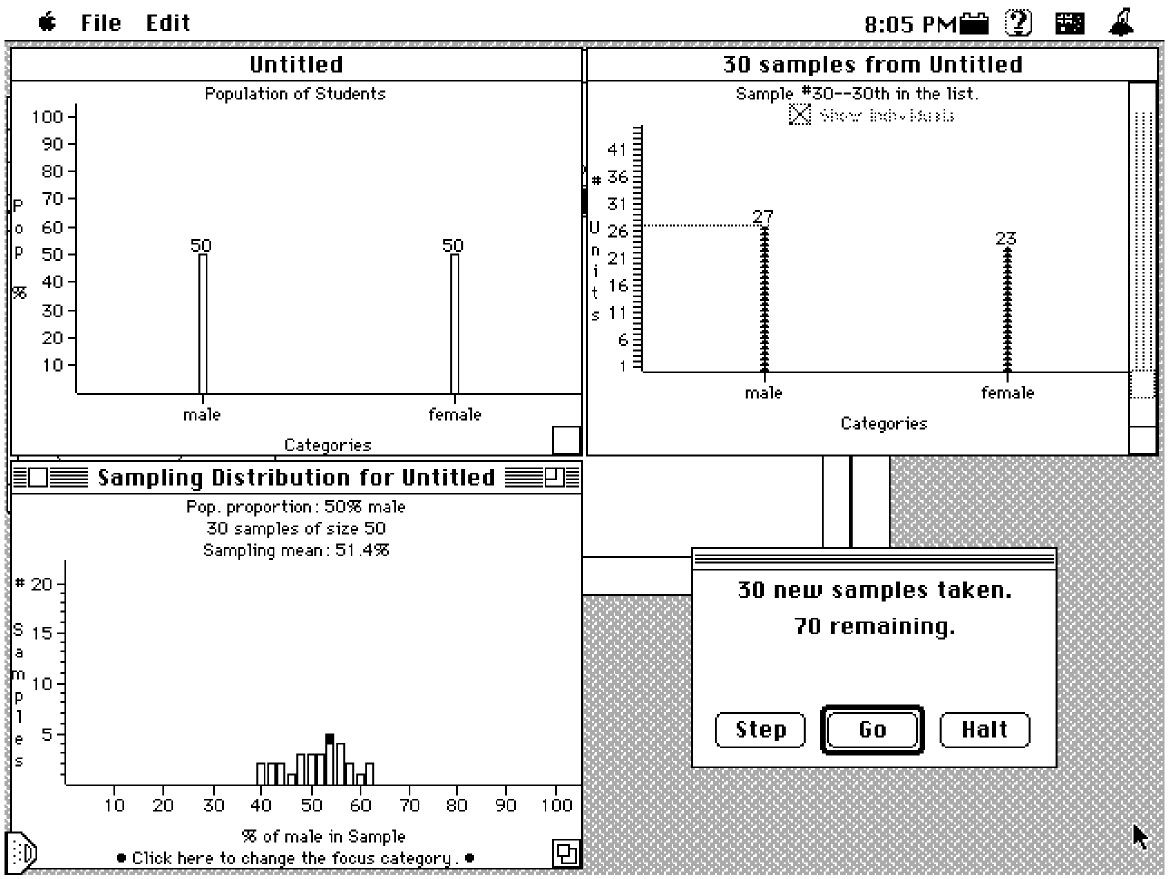

Figure 2: Screen from Sampling Laboratory

The data discussed in this paper was gathered over one three-hour session. In the first hour, students attended a class given in lecture format in which they were introduced to the idea of a sampling distribution. This class included a "hands-on" sampling exercise in which a shovel and a box of beads (white and colored) were passed around the group, and a histogram of the values of the sample proportion obtained by each student was constructed as the sampling proceeded. During this exercise, the population parameter, which is constant at a given point in time for a given population, and the sample statistic, which varied from sample to sample, were emphasized. Also at this time, the accepted notation was discussed, and the terms sampling variablilty and sampling distribution were introduced.

Following this sampling activity, the students were told that they would be using the computer to allow them to investigate further the sampling distribution for the sample proportion. The students were then divided at random into two groups and separated into two different classrooms. Group 1 worked through their simulation exercise using Minitab; Group 2 used Sampling Laboratory. Each group worked on exercises that were as similar as possible; that is, each exercise required the students to generate sampling distributions for samples of different sizes (10, 25, 50, and 100) and to fill in Table 1.

Table 1: Table for recording simulation results

Sample Size	Shape	Center	Spread
n=10			
n=25			
n=50			
n=100			

The students were asked at the completion of the exercise to use the completed table to answer the following focus questions:

- Look at where each of the sampling distributions is centered. How does the center of the sampling distribution appear to relate to the population proportion?
- Look at the spread for each of the sampling distributions. How does the sample size appear to relate to the spread of the sampling distribution?

MEASUREMENT OF UNDERSTANDING

Those of us who teach statistics are well aware that many students seem to be able to learn to conduct routine statistical tasks, but at the same time seem to be unable to demonstrate any understanding of the statistical concepts that underlie these tasks (e.g., Garfield & Ahlgren, 1988; Lipson, 1995). That is, students may exhibit some *procedural* knowledge of the task, but not have the necessary *conceptual* knowledge. This creates a problem when students want to apply their knowledge to a different situation. In general terms, procedural knowledge describes the student’s ability to conduct routine tasks successfully, whereas conceptual knowledge implies an understanding of what is being done and why. Hiebert and Carpenter (1992) provided more precise definitions of procedural and conceptual knowledge by relating them to a student’s cognitive structure:

> Conceptual knowledge is equated with connected networks...A unit of conceptual knowledge is not stored as an isolated piece of information; it is conceptual knowledge only if it is part of a network. On the other hand, we define procedural knowledge as a sequence of actions. The minimal connections needed to create internal representations of a procedure are connections between succeeding actions in the procedure. (p. 78)

Thus, if we are to accept the Hiebert and Carpenter definition of conceptual knowledge, researchers concerned with measuring conceptual knowledge need ways of evaluating the development of the students' cognitive structures, particularly in response to specific educational strategies. In assessing understanding, we are concerned not only with knowledge of the relevant concepts but also with the relationships between these concepts. A method that would appear to be relevant to the current situation is the concept map developed by Novak and Gowin (1984), which has been used with some success in science education for this purpose (e.g., Wallace & Mintzes, 1990).

The concept map

Novak and Gowin (1984) state that "a concept map is a schematic device for representing a set of concept meanings embedded in a framework of propositions" (p. 15). Essentially, constructing a concept map requires a student to identify important concepts concerned with the topic, rank these hierarchically, order them logically, and recognise cross links where they occur. There are many uses for the concept mapping technique that are well-documented in science education (e.g., Peterson & Treagust, 1989; Starr & Krajcik, 1990). Among other things, concept maps have been used by teachers in curriculum planning to determine the content and structure of courses, as an assessment tool to provide insight into the students' understanding, and as a teaching tool to facilitate the development of student understanding (in much the same way as the usefulness of preparing a summary).

Note that it is not the purpose of this paper to validate the use of the concept map as an instrument to measure understanding in statistics. In an earlier study (Lipson, 1995), a significant correlation was found between a concept mapping task and performance on several other tasks. These tasks included the solution of a standard statistical problem, the explanation of a statistical hypothesis test in plain language, and the application of the principals of hypothesis testing to an unfamiliar problem. The results of this study indicated that the concept map provides a measure of both the student's procedural and conceptual knowledge, and was the only instrument investigated that successfully did this. Because it appears to simultaneously assess both a student's procedural and conceptual understanding, the concept map is a useful tool for assessing understanding in statistics. Concept mapping can also be used to explore changes in a student's understanding by examining how student concept maps alter during an instructional sequence. This was how they were used in the current study.

Method

The students enrolled in this course had been instructed in the use of concept maps starting from the beginning of the semester (approximately seven weeks) and had been asked to prepare several concept maps before the experimental session. During the teaching session, the students were asked to construct a concept map before the computer session, but after the sampling activity that was described above.

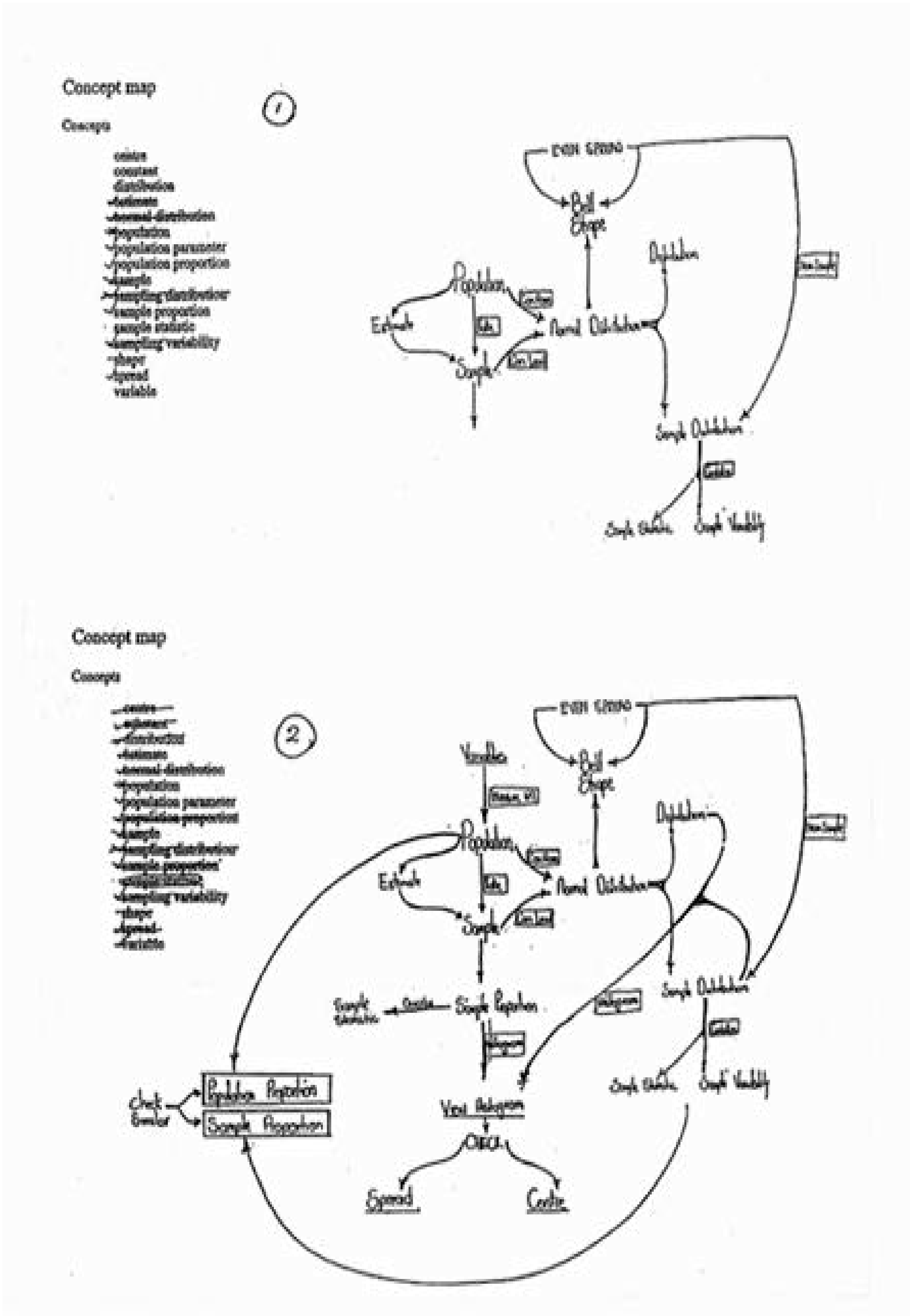

Figure 3: Concept maps constructed by a student before and after computer session

To assist the students in the construction of the concept map, the following terms were supplied: *center, constant, distribution, estimate, normal distribution, population, population parameter, population proportion, sample, sampling distribution, sample proportion, sample statistic, sampling variability, shape, spread, variable*. The list of words was chosen based on the objectives of the teaching sequence, but also to incorporate some of the key pedagogic features of the software. Students were instructed that these words were merely a suggested list and that any words could be omitted or others added. This is the same way that the maps had been used in the past.

The words were listed down the left hand side of a large sheet of paper, and the students were requested to construct their map using pencil only. When the students had completed the maps, they were collected and photocopied. After the completion of the computer sessions, the maps were returned to the students, who were requested to modify their maps in any way they felt was appropriate. The resulting maps were again recorded. Figure 3 shows an example of one student's maps before and after the computer session.

Interpretation of the concept maps

There are many ways a concept map may be interpreted. The original way, as proposed by Novak and Gowin (1984) , involves a scoring system where points are allocated on the basis of the number of concepts involved, the number of levels in the hierarchy used to map the concepts, and the number of cross links that are made between separate strands of the concept map. Although the original method has been successful for certain purposes, it was believed that it would be an inappropriate method in a situation in which the students are given a list of terms to use and asked to add structure to them. The relevant issues here were how the terms had been linked together, whether or nor the linkages were correct, and if important links had been recognized. An alternative technique for evaluating concept maps, which was used here, has been suggested by Kirkwood, Symington, Taylor, and Weiskopf (1994). This approach is based on the identification of a number of key propositions that have been identified by consultation with "experts." The propositions considered "key" to this particular topic were developed by the author in conjunction with colleagues, and are listed in Table 2.

Table 2: Key propositions looked for in the students' concept maps

	Key Propositions
A	Populations give rise to samples.
B	A population has a distribution.
C	Population (distributions) are described by parameters.
D	Parameters are constant.
E	Sample distributions are described by statistics.
F	Statistics are variable, have a distribution.
G	The sampling distribution of $\hat{p}$ is approximately normal.
H	The sampling distribution of $\hat{p}$ is characterised by shape, centre, spread.
I	The spread of the sampling distribution is related to the sample size.
J	The sample statistic can be used to estimate the population parameter.

Results of the concept map analysis

For each student, the following information was recorded: the computer exercise they undertook (M = Minitab, S = Sampling Laboratory), the propositions that were present in their first map (A-J, see Table 2), the propositions that were present in their second map, and any change in the number of propositions included. The results are summarized in Table 3, which shows that the number of propositions added to the students' concept maps during the experiment varied considerably. To give some indication of the extent to which the students changed their maps, a frequency distribution of the number of new propositions the students added to their second map was constructed. Table 4 shows that seven students included three or more new propositions, but 13 students did not change their maps at all.

The frequency with which different propositions were included in the students' concept maps was also of interest. Table 5 summarizes the number of students including each proposition before and after the computer sessions, for each computer-based strategy, and the total over both computer groups.

Table 3: Number of propositions identified in students' concept maps before and after computer sessions

Student	M/S	Map 1	Map 2	Change	Student	M/S	Map 1	Map 2	Change
1	M	3	3	0	17	S	4	4	0
2	M	4	4	0	18	S	3	4	1
3	M	2	2	0	19	S	5	9	4
4	M	4	6	2	20	S	7	7	0
5	M	4	5	1	21	S	2	7	5
6	M	6	6	0	22	S	0	0	0
7	M	0	5	5	23	S	4	6	2
8	M	3	3	0	24	S	4	5	1
9	M	4	4	0	25	S	2	4	2
10	M	2	6	4	26	S	4	6	2
11	M	7	7	0	27	S	6	6	0
12	M	5	7	2	28	S	3	6	3
13	M	4	5	1	29	S	2	4	2
14	M	6	7	1	30	S	7	7	0
15	M	3	6	3	31	S	4	4	0
16	M	7	9	2					

Table 4: Frequency distribution for number of propositions added to the second concept map
(Total number of students = 31)

Number of propositions	0	1	2	3	4	5
Frequency	13	4	7	2	3	2

From Table 5 the following observations can be made:

- Propositions A (Populations give rise to samples) and C (Population distributions are described by parameters) seem to have been assimilated by most students before the computer-based exercises were undertaken.

- Many of the propositions that seem paramount in an *a priori* analysis to an understanding of sampling distribution do not seem to have been evoked by the computer sessions, even though the sessions had been specifically designed with these in mind and students were led to these propositions by the focus questions. This would seem to be particularly true regarding Propositions B (A population has a distribution), D (Parameters are constant), and I (The spread of the sampling distribution is related to the sample size).

- The most common addition to the students' concept maps was Proposition H (The sampling distribution of $\hat{p}$ is characterised by shape, centre, spread), which was added by 13 students. This concept is important in the development of the concept of sampling distribution and for subsequent principals of statistical inference.

Careful analysis of the concept maps allowed subtle, but important, developments in understanding to be identified [e.g., Proposition J--that the sample statistic (proportion) can be used to estimate the population parameter (proportion)]. During the concept map analysis, it became evident that Proposition J could be validly included in the map in two different ways. Most students who included Proposition J did so by directly linking the terms *sample statistic* and *population parameter*, as shown in the map in Figure 4a. Alternatively, the map given in Figure 4b shows an increasing depth of understanding of the relationship between the population parameter and the sample statistic, with the population parameter linked to the features of the sampling distribution (i.e., *shape*, *center*, and *spread*). This notion, that the sampling distribution of the sample proportion is centered at the population parameter, indicates that the student has made links that one could justifiably expect to be important when developing further the ideas of statistical inference.

Table 5: Frequency tables of propositions identified in concept maps before and after computer sessions
(Total number of students = 31)

	Proposition									
Minitab	A	B	C	D	E	F	G	H	I	J
Before	11	2	12	2	9	8	7	4	-	6
After	12	2	15	3	10	9	7	9	3	9
Change	**1**	**-**	**3**	**1**	**1**	**1**	**-**	**5**	**3**	**3**

	Proposition									
Samp Lab	A	B	C	D	E	F	G	H	I	J
Before	13	3	12	1	10	6	4	2	-	6
After	12	4	14	2	12	8	8	10	-	9
Change	**-1**	**1**	**2**	**1**	**2**	**3**	**4**	**8**	**-**	**3**

	Proposition									
Total	A	B	C	D	E	F	G	H	I	J
Before	24	5	24	3	19	14	11	6	-	12
After	24	6	29	5	22	17	15	19	3	18
Change	**1-1**	**1**	**5**	**2**	**3**	**3**	**4**	**13**	**3**	**6**

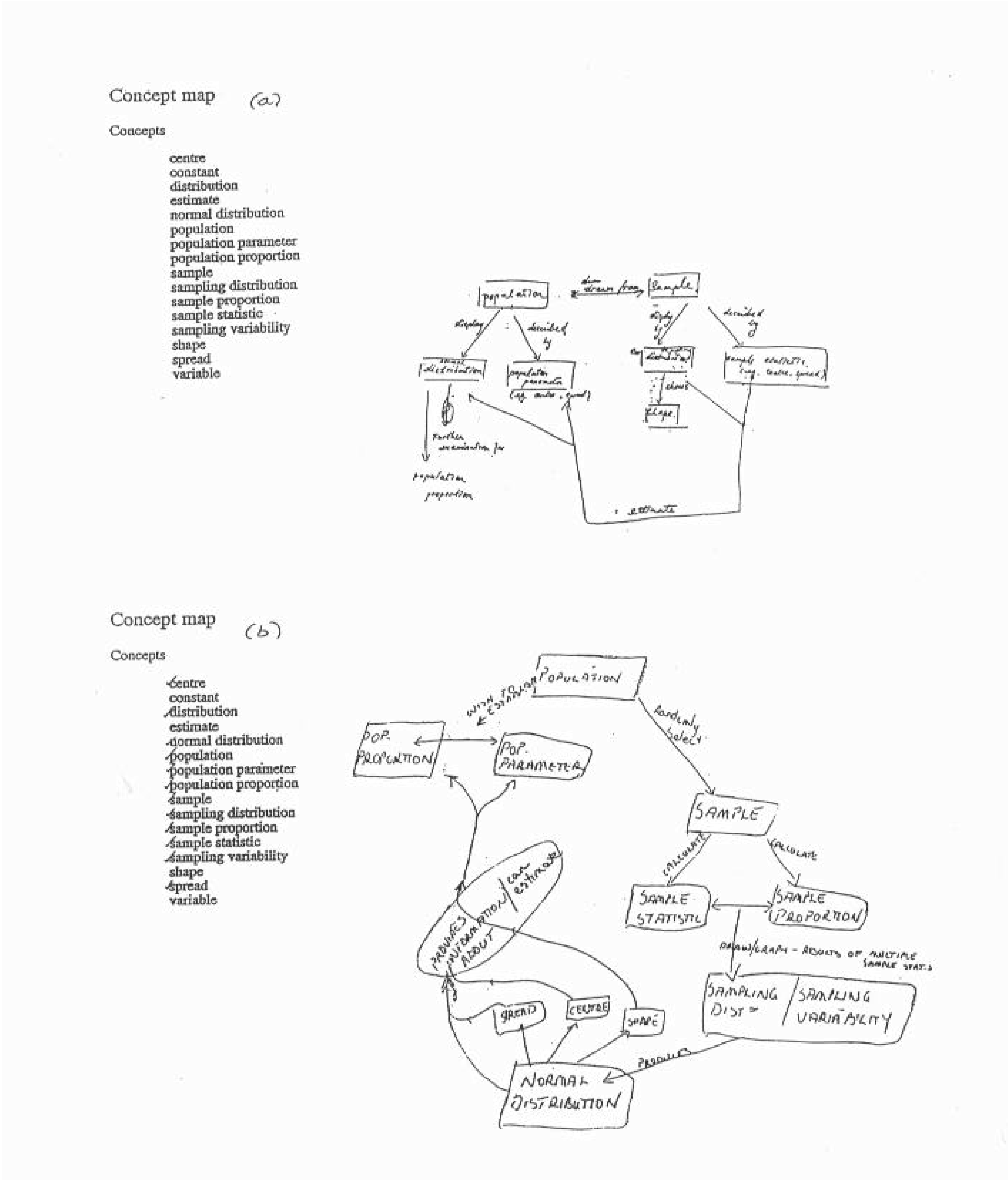

Figure 4a: A concept map where population parameter and sample statistic are directly linked
Figure 4b: A concept map where the population parameter is related to the sampling distribution

DISCUSSION AND CONCLUSIONS

This study compared two computer-based strategies (Minitab and Sampling Laboratory) for facilitating understanding of the sampling distribution of the sample proportion. The results indicate that they were both equally effective in achieving the instructional objectives, which is contrary to theoretical expectations.

The reason for the lack of difference between the two strategies might be because there really is no difference between them in these circumstances. This possibility should not be disregarded, and in fact could be the case if one considers the total teaching experience in which the computer activities are embedded. The participation in the sampling activity that formed the common introduction to the computer simulations should be considered to be a critical component of concept development. It is worth noting that before either of the computer simulation sessions the median number of concepts contained in the maps for both groups was 4. This indicates that the students had already begun to appreciate some of the key ideas of the sampling distribution, which were then reaffirmed, and extended, by the computer simulation experience.

A theoretical explanation of this situation may be given by consideration of the *zone of proximal development* (Vygotsky, 1978). This theory has been interpreted by Brown, Ash, Rutherford, Nakagana, Gordon, and Campione (1993) as follows:

> It defines the distance between current levels of comprehension and levels that can be accompished in collaboration with people or powerful artifacts. The zone of proximal development embodies a concept of readiness to learn that emphasizes upper levels of competance. (p. 191)

In line with this theory, one could speculate that by experiencing the “hands-on” sampling exercise first, students who subsequently conducted the computer simulation exercise using Minitab, which makes less obvious connections between computer simulation and the real world than Sampling Laboratory, were able to make these connections themselves. This would explain why, although the simulations carried out using Sampling Laboratory are much more explicitly related to the real world sampling situation, there was little difference in the effectiveness of the two programs. This would also suggest that the difference between the computer strategies might be evidenced in a different experiment, where the “hands-on” sampling exercise was not part of the total experience.

The effectiveness of the computer simulation sessions cannot be determined directly from this experiment. Certainly, the number of concepts exhibited by the maps was larger after the computer exercises ($\underline{M} = 5.3$) than before ($\underline{M} = 4.0$), and this difference was significant [$t(31) = -4.94$, $p < 0.00005$). However, the lack of a control group in this experiment means we cannot attribute the increase in the number of key concepts to the computer sessions; however, this should be the subject of further studies.

A further reason for the lack of a significant difference between the two computer-based strategies may have been the inadequate time allowed for students to fully complete all activities, as well as to complete their maps. Because this course was taught in a three-hour block during the evenings, many students may well have felt that the effort required to modify their map was too much. This suspicion may be further supported by the fact that 13 of the 31 students did not modify their maps at all. Future replication of the study will endeavor to ensure that such problems do not occur again.

Notwithstanding this, certain conclusions can be drawn on the basis of the study. The results show that participating in a computer simulation activity was associated with a development of understanding for many students: 18 out of 31 (58%) students added one or more propositions to their maps, and 7 of the 31 (23%) students added three or more. After the simulation exercises, 23 of the 31 students had included at least 4 of the propositions on their concept maps. The concept maps allowed the investigator to observe subtle growth in the students' conceptual knowledge.

The understanding of a concept exhibited by a student will often be incomplete or faulty; however, by using the concept map even the most minor development could be observed and identified. Each student had a unique concept map, and every person changed their map in different ways. This individual evolution of knowledge is consistent with the theory that understanding is constructed by the student on the basis of their current cognitive structure, rather than received from an instructor on an information transfer basis.

Finally, the concept maps also seem to indicate that certain propositions are well established with students while others, seemingly just as important, are not. Why some of the concepts that the activities are specifically designed to illustrate are not established is not clear from this study. Perhaps the concepts that appear important to educators on the basis of an *a priori* analysis are not fundamental to an overall understanding of statistical inference, or perhaps there were deficiencies in the instructional sequence. This clearly needs to be the concern of future investigations.

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