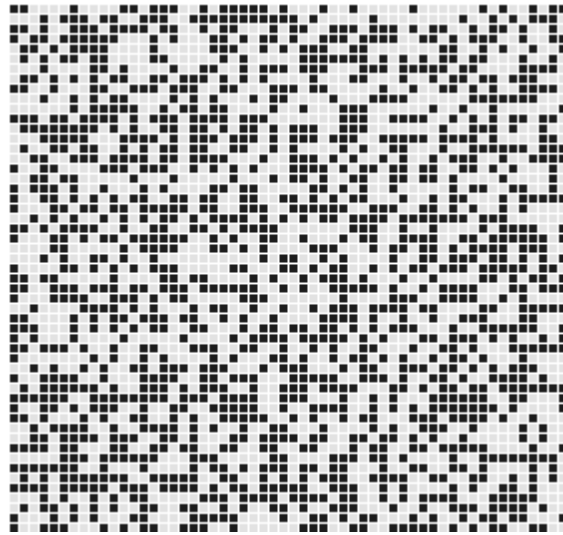## Bathroom Tiles Problem

The Question

The restrooms at the convention center in Indianapolis have one inch square blue and white tiles on the floor, laid out in an apparently random pattern similar to the one at right. The room is approximately square, 24 feet on a side.

The question is, is the observed pattern actually random?

One approach is to count the longest "run" of tiles of the same color, which we found to be 17, and compare that with what one would expect.

There are about 288 tiles on a side, so, counting both rows and columns, 576 lines of tiles. (Are we really justified in considering the rows and columns as independent of each other? We think that when the probability of getting either color is about 0.5, the assumption works well enough.) If the probability of each color is 0.5, independent of adjoining tiles, what is the chance of getting 17 tiles of the same color in a row?

The Simulation

We constructed a simulation of the bathroom floor in Fathom.

The collection Tiles contains 288 cases, each with a color, randomly chosen. It simulates one row or column in the floor.

| Attribute	Value	Formula
color	blue	randomPick ("blue", "white")
runSoFar	1	runLength (color)
runLength	1	if (next (runSoFar) = 1){runSoFar

Each tile has three attributes computed using the formulas shown at right. The table (below right) shows the first 10 tiles in a typical collection and the histogram (below left) shows a typical distribution of run lengths in one collection. Notice that the maximum of these run lengths for this particular collection is eight.
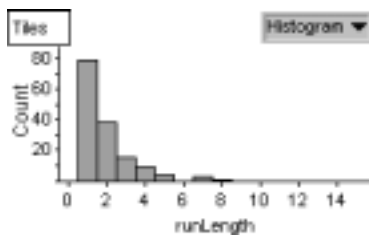
We compute the maximum run length in Tiles as a measure, allowing us to collect this measures from Tiles 576 times, simulating the 576 rows and columns.

| color	runSoFar	runLen...
white	1	
white	2	2
blue	1	
blue	2	2
white	1	1
blue	1	
blue	2	
blue	3	
blue	4	
blue	5	5

The distribution of one set of 576 maximum run lengths is shown in the histogram for the "Measures from Tiles" collection. (Think of each "Measures from Tiles" collection as one restroom floor.) Notice that in this particular collection of measures, the maximum of maxRunLength was 21.
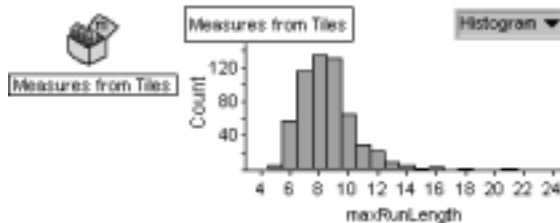
Since the particular bathroom floor we examined

had a maximum run length of 17, the question is how likely is it that the maximum run length over the entire floor is 17. We compute as a measure, the maximum of all 576 maximum run lengths. This allows us to collect these measures and look at the distribution of the "floor maximum run length."

The histogram at right shows the results for 100 floors. We see that 17 occurred 33 times and is at the center of the distribution, lending strong support to the idea that the tiles on the floor are laid out in a truly random pattern.
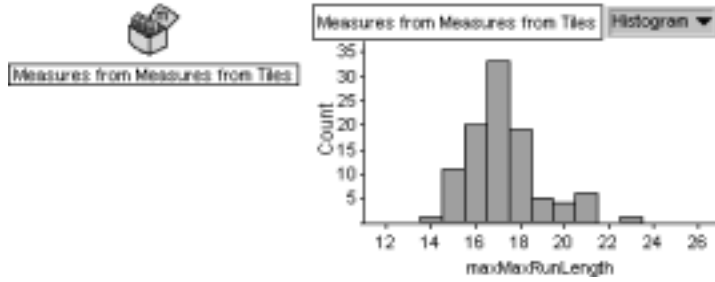
Bill Finzer and Laurie Snell

ASA convention, Indianapolis, August, 2000